Blocking in All-Optical Networks

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Abstract— **We present a new analytical technique, based on the inclusion–exclusion principle from combinatorics, for the analysis of all-optical networks with no wavelength conversion and random wavelength assignment. We use this technique to propose two models of low complexity for analysing networks with arbitrary topologies and traffic patterns. The first model improves the current technique by Birman [5] in that the complexity of calculation is** *independent* **of hop-length and scales only with the capacity of the link as against that of [5] which grows exponentially with hop-length. We then propose a new heuristic to account for wavelength correlation and show that the second model is accurate even for sparse networks. Our technique can also be extended to analyse Fixed Alternate and Least Loaded Routing.**

I. Introduction

Wavelength division multiplexing is a promising technology which, in conjunction with wavelength routing, can make optical networks with hundreds of nodes and throughput of the order of Gbs/sec per node practical in the near future. This is because wavelength routed all-optical networks offer wavelength reuse and remove the electro-optic bottleneck. In this work we consider circuit switched all-optical networks since they are a natural outcome of current WDM technology [1]. Call requests arrive at random and are assigned a free wavelength (if available) on each link of the path they use for the duration of the call. If the nodes have wavelength conversion capability, the call can be assigned different wavelengths on each link of the path used. In such a situation, the all-optical network reduces to a conventional circuit switched network. However, if the nodes cannot perform wavelength conversion, the call must be assigned the *same* wavelength on *all* the links of the path used. This is known as the *wavelength continuity constraint* and makes networking in the all-optical domain significantly different from conventional circuit switched networks. Networks with wavelength changers have a lower call blocking probability compared to those without because they can accept a call if a wavelength (which can be different) is free on each link of the path, whereas networks without changers require the *same* wavelength to be free on all the links of a path in order to honor a call. Wavelength converters are still in the experimental stage and are likely to remain expensive, if implemented. Hence it is important to quantify the call blocking performance of optical networks without wavelength conversion to verify the effectiveness of wavelength changers.

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Research has shown that the *wavelength continuity constraint* introduces load correlation between links, and that the blocking in the network is affected not only by the routing scheme used but also by the choice of a wavelength assignment scheme. A bound on the carried traffic in an arbitrary network by *any* Routing and Wavelength Assignment algorithm (RWA) algorithm was derived in [2]. The bound is however only asymptotically achievable. Various schemes that combine the wavelength assignment and routing problem have been proposed and studied through simulations in the literature, for example [8], [9] and [14]. Analytical models for the first fit wavelength assignment scheme have been proposed in [8]- [10]. They however use versions of the overflow traffic model and are applicable only when the number of wavelengths is small (4 to 8). Least Loaded Routing has been studied in [13] and Fixed Alternate Routing in [14]. The reader is referred to [12] for a review of these schemes and their effectiveness.

Analytical models for analyzing the performance of optical networks with fixed routing, random wavelength assignment and without wavelength conversion have been proposed in [4]-[6]. In [4], Barry et al. proposed an analytical model to study the effectiveness of wavelength changers, taking wavelength correlation into account. However the model does not take into account the dynamic nature of the traffic. The model proposed by Subramanium et al. [6] takes both dynamic traffic and wavelength correlation into account and has been shown to be accurate even for sparse networks like rings. Moreover, the model has a moderate complexity. It is however applicable in the strict sense only to networks with uniform traffic and regular topologies. In case of irregular topologies and traffic distributions, only ensembles like the average degree of a node are used. Another model proposed by Birman [5] uses a reduced load approximation approach with state-dependent arrival rates. The model is shown to be good for small networks where multi-link traffic is not appreciable and is applicable to arbitrary topologies and traffic patterns. It is however computationally intensive, with the complexity growing exponentially with the number of hops. It also ignores the load correlation between links due to the continuity constraint. Hence it is tractable only for small, dense networks. The reader is referred to [11] and [12] for a review of these analytical models.

Future wide area networks are most likely to be all-optical networks with tens, if not hundreds of nodes, connected in an arbitrary fashion. In such a situation, the models in the current literature would not apply satisfactorily to the analysis of the network in question. Thus our goal is two-fold. We require a technique applicable to arbitrary topologies which is computationally tractable, and also gives reasonable estimates of blocking probabilities for design purposes and the analytical study of issues like benefits of wavelength changers, alternate routing and so on. With these goals in mind we propose two techniques in this paper which reduce the complexity of calculation considerably and are applicable to arbitrary topologies and traffic patterns. The first technique makes the same assumptions as in [5] and is called the *Independence Model*. Estimates of the blocking probability from this model are reasonable when the network is dense or end-to-end traffic is negligible, but this model overestimates significantly when the multi-hop calls have appreciable traffic and the network is sparse. In fact, we show that the estimates are identical to those obtained in [5], but without the bottleneck of exponential complexity in hop-length. The complexity scales only with the capacity of the link and is *independent* of hop-length. To account for wavelength correlation we propose another model which we call, naturally, the *Correlation Model*. We show that this model is accurate even for sparse networks.

The rest of the paper is organized as follows. In Section II we outline the basic network and traffic assumptions used throughout the paper. We present the Independence Model in Section III and the Correlation Model in Section IV. We present our results in Section V and conclude in Section VII.

II. Network and Traffic Model assumptions

In this section we state the assumptions about the network and traffic that are used in our models for calculating the path blocking probability of an optical network with no wavelength changers. All assumptions stated here are valid throughout the paper.

A. *Assumptions regarding the Network and Offered Traffic*

1. The network consists of J links connected in an arbitrary fashion.
2. Each link has the same C wavelengths
3. Calls for a node pair S arrive according to a Poisson process with rate λ_S
4. The duration of each call is exponentially distributed with unit mean.
5. A call can be accommodated on a route only if the *same* wavelength is free on all the links of the selected route. If there is no such wavelength free, the call is blocked and lost.
6. The wavelength assigned to a route is chosen randomly from the set of free wavelengths. This assumption makes all wavelengths identical and the analysis tractable.

B. *Traffic Model*

We assume that the idle wavelength distribution on a link can be described by the state dependent routing model first proposed by Kelly and developed in [7]. The same model is also used in [5]. Below, we describe the model and its assumptions.
Let X_R be the random variable representing the number of free wavelengths on route R .
Let X_j be the random variable representing the number of free wavelengths on link j. Let

$$q_j(m) = Pr\{X_j = m\}$$

be the probability that **exactly** m wavelengths are free on link j. The random variables X_j are assumed to be independent, that is

$$q(\mathbf{m}) = \prod_{j=1}^{J} q_j(m_j). \tag{1}$$

Following [7] we assume that, given exactly m idle wavelengths on link j, the time until the next call setup on j is exponentially distributed with parameter $\alpha_j(m)$. It then follows that the number of idle wavelengths on link j can be modelled as the outcome of a birth-death process. We then have

$$q_j(m) = \frac{C(C-1)...(C-m+1)}{\alpha_j(1)\alpha_j(2)...\alpha_j(m)} q_j(0) \tag{2}$$

where

$$q_j(0) = \Big[1 + \sum_{m=1}^{C} \frac{C(C-1)....(C-m+1)}{\alpha_j(1)\alpha_j(2)....\alpha_j(m)}\Big]^{-1}. \tag{3}$$

The call set up rate is a function of the routing scheme used. We assume fixed routing in this section and the next two sections also. This means that each node-pair has exactly one pre-determined route. If an arriving call does not find a free wavelength on this route, it is blocked and lost.

For fixed routing the call setup rate when there are exactly m idle wavelengths on link j, $\alpha_j(m)$ is obtained by combining the contributions from all the request streams that have link j as their member. This expression was first obtained for an all-optical network in [5].

$$\begin{aligned}\alpha_j(m) &= 0 \quad \text{if} \quad m = 0 \\ &= \sum_{R:j\in R} \lambda_R \Pr\{X_R > 0 \mid X_j = m\} \\ & \quad m = 1, 2, \ldots, C. \end{aligned} \tag{4}$$

Typically, in a network, the blocking probabilities and arrival rates to a link are coupled to each other by the fact that the blocking determines the traffic carried by the network and the carried traffic in turn determines the blocking. This leads to a set of coupled non-linear equations which must be solved to obtain the blocking probabilities. The usual scheme implemented in most analyses including ours is solution by iteration.

III. The Independence Model

In this section we present a technique for calculating the blocking probability along a path using the link distributions. The probability that a call traversing a route R consisting of a single link j (say) is blocked is simply given by

$$B_R = q_j(0) \tag{5}$$

which is the probability that there is no idle wavelength on link j. In order to calculate the blocking probability for a multi-hop path we introduce the following random variable:

Let $Y_{i,j}$ be the random variable denoting the state of wavelength i on link j. Define

$$\begin{aligned} Y_{i,j} &= 0, \quad \text{if wavelength } i \text{ is free on link } j, \\ Y_{i,j} &= 1, \quad \text{if wavelength } i \text{ is used on link } j. \end{aligned}$$

From the assumption of random wavelength assignment, we then have that the the probability that a *fixed* set of i wavelengths is free on some link j is

$$Pr\{Y_{1,j} = 0, \ldots, Y_{i,j} = 0\} = \sum_{m=i}^{C} q_j(m) \frac{\binom{m}{i}}{\binom{C}{i}}. \tag{6}$$

Note that since the wavelengths are identical by virtue of the assumption of random assignment, all sets of i wavelengths are equally likely to be free.

Denote by

$$\beta_{i,j} = Pr\{Y_{1,j} = 0, Y_{2,j} = 0, \ldots, Y_{i,j} = 0\} \tag{7}$$

the probability that a fixed set of i wavelengths is free on link j.

Now, the probability that a multi hop-route R (say) is blocked is the probability that there is no wavelength which is free on all the links used by R. We have

$$B_R = \Pr\{X_R = 0\} = 1 - Pr\{X_R > 0\}.$$

Let g_i^R be the probability that a *fixed* set of i wavelengths is free on the route R. Then, from the inclusion-exclusion principle and the assumption of random wavelength assignment, it follows that

$$\Pr\{X_R > 0\} = \sum_{i=1}^{C} (-1)^{i-1} \binom{C}{i} g_i^R.$$

For notational convenience we describe in detail the method of calculation of g_i^R for a two link path consisting of links A and B. The method is readily generalized for paths with higher hop-lengths. For the two link path

$$\begin{aligned} g_i^R &= \Pr\{(Y_{1,A} = 0, Y_{1,B} = 0), (Y_{2,A} = 0, Y_{2,B} = 0), \\ & \quad \ldots (Y_{i,A} = 0, Y_{i,B} = 0)\}. \end{aligned}$$

Using the assumption that the sets of wavelengths on links are independent (1), we have

$$\begin{aligned} g_i^R &= \Pr\{Y_{1,A} = 0, Y_{2,A} = 0, \ldots, Y_{i,A} = 0\} \cdot \\ & \quad \Pr\{Y_{1,B} = 0, Y_{2,B} = 0, \ldots, Y_{i,B} = 0\}, \end{aligned}$$

or, from (7)

$$g_i^R = \beta_{i,A} . \beta_{i,B}.$$

The method described above immediately generalizes to higher hop lengths. The probability that a call is blocked on a H–hop route R is given by

$$B_R = \Pr\{X_R = 0\} = 1 - \Pr\{X_R > 0\} \tag{8}$$

where

$$\Pr\{X_R > 0\} = \sum_{i=1}^{C} (-1)^{i-1} \binom{C}{i} g_i^R \tag{9}$$

and g_i^R is given by

$$g_i^R = \prod_{j:j\in R} \beta_{i,j}. \tag{10}$$

A. Calculation of State Dependent Arrival Rates

The arrival rate of a request stream from route R to link j, given that there are m wavelengths free on link j, is given by (4) as

$$\begin{aligned}\alpha_j^R(m) &= 0 \qquad \text{if m=0}\\ &= \lambda_R.\, \Pr\{X_R > 0 \mid X_j = m\}\\ & \quad m = 1, 2, \ldots, C\end{aligned}$$

where

$$\alpha_j(m) = \sum_{j:j\in R} \alpha_j^R(m).$$

If the route consists of a single link then the probability term is clearly 1 for $m \neq 0$. The term for a multi-hop path may be calculated as follows.

$$Pr\{X_R > 0 \mid X_j = m\} = \sum_{i=1}^{m} (-1)^{i-1} \binom{C}{i} g_i^R(X_j = m) \tag{11}$$

where $g_i^R(X_j = m)$ is the conditional probability that a *fixed* set of i wavelengths is free on the route R given exactly m wavelengths are free on link j. It may be calculated as

$$g_i^R(X_j = m) = \prod_{k:k\in R, k\neq j} \beta_{i,k} \frac{\binom{m}{i}}{\binom{C}{i}} \tag{12}$$

because

$$Pr\{Y_{1,j} = 0, \ldots, Y_{i,j} = 0 \mid X_j = m\} = \frac{\binom{m}{i}}{\binom{C}{i}}.$$

Note that the summation in (11) runs only up to m since m is an upper bound on the number of free wavelengths on the path.

B. Algorithm for Computation of Blocking Probabilities in the network

As mentioned in Section II, we need to solve a set of non-linear coupled equations to obtain the blocking probabilities. Though we have not been able to prove that a fixed point exists for this system of coupled equations (and if it does whether it is unique), in practice the method of solution by repeated substitution converges in a few iterations for a variety of topologies. The method of repeated substitution to solve for the blocking probabilities may be implemented as follows:

Let B_R be the probability that a call for route R is blocked.
1. For all routes R initialize $\hat{B_R}$ to zero. For $j = 1, \ldots, J$ initialize $\alpha_j(0) = 0$ and
$\alpha_j(m) = \sum_{R:j\in R} \lambda_R, m = 1, \ldots C.$
2. Determine the idle capacity distribution of all links $q_j(.)$, $j = 1, \ldots, J$ using (2) and (3).
3. Calculate $\beta_{\mathbf{j},\mathbf{m}}$ for all links, $j = 1, \ldots, J$ and $m = 1, \ldots, C$ using (6)
4. Calculate $\alpha_j(.)$, $j = 1, \ldots, J$ using (4), (11) and (12)
5. Calculate B_R for all routes using (5) if it is a single link and (8) and (9) and (10) for a multi-hop path.
If $\max_R \mid B_R - \hat{B_R} \mid < \epsilon$ then terminate. Else let $\hat{B_R} = B_R$ and go to step 2.

IV. The Correlation Model

As will be shown in Section V the Independence model presented in the previous section gives good results for dense networks but overestimates the blocking probability significantly for sparse networks like rings. This is because it does not account for load correlation introduced by the wavelength continuity constraint between adjacent links. That is, it assumes that sets of wavelengths on adjacent links are independent, which is not a good assumption when the network is sparse ([4], [6]). In sparse networks like rings, the number of choices for a route is small. Hence calls tend to stay together over a longer set of links leading to increased correlation since they use the same wavelength on all the links.

In this section we extend the Independence model to take this correlation into account. The tradeoff for accuracy however, as will be shown later, is that the complexity of calculation increases. But we will see that it is still much less than that of the model in [5].

The network and traffic assumptions made in Section II remain the same. All the notations and variables used in the previous section retain their original meanings. We also assume that the parameters $\beta_{i,j}$ may be calculated as before, using (6). The point of departure from the previous model is that we no longer make the assumption of unconditional wavelength independence made previously while calculating the blocking on a multi-hop path R. Throughout this analysis we assume that the links on a route are ordered and the direction of a call is fixed a priori.

To cater for link correlation we make a set of assumptions as described below:

- **A1.** The state of a wavelength i on link j is independent of the state of some *other* wavelength k on link $j-1$, given the state of the same wavelength i on link $j-1$, or the state of wavelength k on the same link j. More formally,

$$Y_{i,j} \coprod Y_{k,j-1} \quad k \neq i \quad \text{given} \quad Y_{k,j} \quad \text{or} \quad Y_{i,j-1}.$$

- **A2.** On a given route, the state of a wavelength on a link j is independent of the state of the *same* wavelength on previous or successive links of the route, given the state of the wavelength on link $j-1$. More formally

$$Y_{i,j} \coprod Y_{i,l} \quad j \neq l \quad \text{given} \quad Y_{i,j-1}.$$

The probability of blocking on a multi-hop route R is given as before by

$$B_R = \Pr\{X_R = 0\} = 1 - \Pr\{X_R > 0\}$$

and

$$Pr\{X_R > 0\} = Pr\{X_R > 0\} = \sum_{i=1}^{C} (-1)^{i-1} \binom{C}{i} g_i^R$$

where the g_i^R s retain their usual meaning.

We now depart from the technique in the previous section in that we derive a new expression for calculating g_i^R. For clarity of exposition we first derive the expression for g_i^R on a two link path. We shall then extend it for higher hop-length paths. Consider a two link path R (say) over links A and B. We have

$$\begin{aligned} g_i^R &= \Pr\{(Y_{1,A} = 0, Y_{1,B} = 0),\ (Y_{2,A} = 0, Y_{2,B} = 0), \\ &\quad \cdots (Y_{i,A} = 0, Y_{i,B} = 0)\}. \end{aligned}$$

Using the chain rule and assumption (**A1**) this may be simplified to

$$\begin{aligned} g_i^R &= \Pr\{Y_{i,A} = 0 \mid Y_{i-1,A} = 0 \ldots Y_{1,A} = 0, Y_{i,B} = 0\} \\ &\quad \ldots \Pr\{Y_{1,A} = 0 \mid Y_{1,B} = 0\} \cdot \beta_{i,B}. \end{aligned} \tag{13}$$

The term $\Pr\{Y_{i,A} = 0 \mid Y_{i-1,A} = 0 \ldots Y_{1,A} = 0, Y_{i,B} = 0\}$ can be further simplified as follows. Define the following new variables

$$\gamma_{j,j-1}^{(0)} = \Pr\{Y_{i,j} = 0 \mid Y_{i,j-1} = 0\} \tag{14}$$

and

$$\gamma_{j,j-1}^{(1)} = \Pr\{Y_{i,j} = 0 \mid Y_{i,j-1} = 1\}. \tag{15}$$

Note that the $\gamma_{j,j-1}^{(.)}$s do not depend on the wavelength index i since all wavelengths are identical. Also define

$$\begin{aligned} \eta_{i,j} &= \beta_{i,j} \quad \text{if} \quad i = 1 \\ &= \frac{\beta_{i,j}}{\beta_{i-1,j}} \quad \text{otherwise}. \end{aligned} \tag{16}$$

Observe that $\eta_{i,j}$ is the conditional probability of wavelength i being free given that $i-1$ other wavelengths are free, i.e.,

$$\eta_{i,j} = \Pr\{Y_{i,j} = 0 \mid Y_{1,j} = 0, Y_{2,j} = 0, \ldots, Y_{i-1,j} = 0\}.$$

The above defined terms along with assumption **(A1)** and Bayes rule allows us to write, after some manipulation:

$$\Pr\{Y_{i,A} i{=} 0 \mid Y_{i-1,A} = 0 \ldots Y_{1,A} = 0, Y_{i,B} = 0\} = \frac{\gamma_{B,A}^{(0)}\ \eta_{i,A}}{\gamma_{B,A}^{(0)}\ \eta_{i,A} + \gamma_{B,A}^{(1)}.(1 - \eta_{i,A})}. \tag{17}$$

Substituting (17) in (13) yields

$$g_i^R = \prod_{k=1}^{i} \frac{\gamma_{B,A}^{(0)}\ \eta_{k,A}}{\gamma_{B,A}^{(0)}\ \eta_{k,A} + \gamma_{B,A}^{(1)}.(1 - \eta_{k,A})}.\beta_{i,B}. \tag{18}$$

A. *Estimation of the Correlation Coefficients*

We have introduced two new parameters $\gamma_{j,j-1}^{(0)}$ and $\gamma_{j,j-1}^{(1)}$ which characterize the load correlation between two adjacent links. The same correlation coefficients were first obtained in [4] and derived again in [15]. We now propose a method to calculate these coefficients for arbitrary topologies and under general traffic patterns.

Let $P_l^{(j)}$ be the probability that a session occupying wavelength λ on link j does not continue to link $j+1$.

Let $P_n^{(j)}$ be the probability that a new call arrives on wavelength λ at link j. A new call on j is one which does not pass through link $j-1$. Then

$$\gamma_{j,j-1}^{(0)} = \Pr\{Y_{\lambda,j} = 0 \mid Y_{\lambda,j-1} = 0\} = (1 - P_n^{(j)}) \tag{19}$$

and

$$\gamma_{j,j-1}^{(1)} = \Pr\{Y_{\lambda,j} = 0 \mid Y_{\lambda,j-1} = 1\} = P_l^{(j-1)}(1 - P_n^{(j)}) \tag{20}$$

This may be explained as follows. $\gamma_{j,j-1}^{(0)}$ is the probability that wavelength λ is free on link j given that it is free on link $j-1$. This is simply the probability that no new call arrives on wavelength λ on link j which by definition is $(1 - P_n^{(j)i})$. $\gamma_{j,j-1}^{(1)}$ is the probability that wavelength λ is free on link j given that it is used on $j-1$. This is the probability that wavelength λ is used on $j-1$ by a session that does not continue to link j and that no new call arrives on wavelength λ on link j which is $P_l^{(j-1)}(1 - P_n^{(j)})$.

Hence the correlation coefficients are actually $P_l^{(j)}$ and $P_n^{(j)}$ which are then substituted in a suitable form to obtain the final expression for g_i^R.

$P_l^{(j)}$ may be calculated as

$$P_l^{(j)} = \frac{\tilde{\lambda}(j, j+1)}{\tilde{\lambda}_j} \tag{21}$$

where

$$\tilde{\lambda}_j = \sum_{m=1}^{C} \alpha_j(m) \cdot q_j(m) \tag{22}$$

is the average arrival rate of traffic to link j, and

$$\tilde{\lambda}(j,k) = \sum_{\substack{\mathbf{R}:j\in R\\ k\notin R}} \sum_{m=1}^{C} \alpha_j^R(m) \cdot q_j(m) \tag{23}$$

is the rate of accepted traffic which passes through link j but does *not* pass through link k.
Thus, we model $P_l^{(j)}$ as the ratio of arrival rate of traffic to link j that does not continue to link $j+1$ to the total traffic arriving at link j which is a reasonable approximation.
We calculate $P_n^{(j)}$ as follows. Let ρ_j be the probability that a wavelength is busy on link j, i.e., ρ_j is a measure of wavelength utilization of link j. Then

$$P_n^{(j)} = \rho_j \frac{\tilde{\lambda}(j,j-1)}{\tilde{\lambda}_j} \tag{24}$$

where ρ_j is given by

$$\rho_i = 1 - \Pr\{Y_{1,j} = 0\} = 1 - \beta_{1,j}. \tag{25}$$

Hence $P_n^{(j)}$ is the probability that the wavelength is in use on link j and the session using it is one that arrives without passing through link $j-1$, i.e., a new session.
Substituting for $\gamma_{j,j-1}^{(0)}$ and $\gamma_{j,j-1}^{(1)}$ from (19) and (20), the expression for g_i^R may be written as

$$g_i^R = \prod_{k=1}^{i} \frac{\eta_{k,A}}{\eta_{k,A} + P_l^{(A)}(1-\eta_{k,A})} \beta_{i,B}. \tag{26}$$

Observe that if we put $P_l^{(A)}$=1 (negligible two-link traffic), g_i^R reduces to the expression obtained in the Independence model presented in the previous section which is correct. Again, if $P_l^{(A)} \to 0$, (negligible single link traffic) the expression reduces to $\beta_{i,B}$, which is also correct. Hence we expect the Correlation Model to perform well under different patterns of traffic, an observation confirmed in Section V. Also observe that the final expression for g_i^R after substituting for $\gamma_{j,j-1}^{(0)}$ and $\gamma_{j,j-1}^{(1)}$ is independent of $P_n^{(j)}$! This comes about because we make the assumption that the probability that a new call arrives on wavelength λ on link j is independent of the state of the wavelength on link $j-1$, that is, $P_n^{(j)}$ is independent of $P_l^{(j)}$.

The expression for blocking probability can now be easily generalized to higher hop paths. For symbolic convenience, let the links on the path be numbered $1, 2, \ldots, H$. Only the expression for g_i^R needs to be modified. This is done as follows. We make use of assumption **(A2)** to divide the path into two link subsections and proceed as before for each two link subsection to obtain the probability of blocking on a multi-hop path as

$$\Pr\{X_R = 0\} = 1 - \Pr\{X_R > 0\} \tag{27}$$

and

$$\Pr\{X_R > 0\} = \sum_{i=1}^{C} (-1)^{i-1} \binom{C}{i} g_i^R \tag{28}$$

where

$$g_i^R = \prod_{j=1}^{H-1} \prod_{k=1}^{i} \frac{\eta_{k,j}}{\eta_{k,j} + P_l^{(j)} \cdot (1-\eta_{k,j})} \beta_{i,H}. \tag{29}$$

B. *Calculation of State Dependent Arrival Rates*

Recall from Section II, that in order to calculate state dependent arrival rates, we need to calculate the probabilities

$$\Pr\{X_R > 0 \mid X_j = m\} = \sum_{i=1}^{m} (-1)^{i-1} \binom{C}{i} g_i^R(X_j = m) \tag{30}$$

where $g_i^R(X_j = m)$ is the conditional probability that a set of i wavelengths is free on the route R given exactly m wavelengths are free on link j . We calculate this by splitting the path into three independent subsections, the path consisting of links before j, link j, and the path consisting of links after j.
Then, $g_i^R(X_j = m)$ is modified to

$$\begin{aligned} g_i^R(X_j = m) &= \prod_{\substack{n=1\\ n\neq j}}^{H-1} \prod_{k=1}^{i} \frac{\eta_{k,n}}{\eta_{k,n} + P_l^{(n)}(1-\eta_{k,n})} . \beta_{i,H} \frac{\binom{m}{i}}{\binom{C}{i}} \\ & \quad \text{if} \quad j \neq H \\ &= \prod_{n=1}^{H-1} \prod_{k=1}^{i} \frac{\eta_{k,n}}{\eta_{k,n} + P_l^{(n)}(1-\eta_{k,n})} . \frac{\binom{m}{i}}{\binom{C}{i}} \\ & \quad \text{if} \quad j = H. \end{aligned} \tag{31}$$

where all symbols retain their usual meaning.

Note that the summation in (30) runs only up to m because m is an upper bound on the number of free wavelengths on the path. The algorithm for calculating blocking in a network using the Correlation Model is similar to that given in the previous section and hence we omit it. We note that the Correlation Model is directional and may not yield the same results if we proceed along the path in the opposite direction. To reduce this effect, we assume that half the traffic on a route is offered in one direction and half in the other. We then calculate the blocking probabilities for the path in each direction and take the average of the two blocking probabilities.

V. Results and Numerical Computation

In this section we analyse the complexity of the techniques presented in this work and also examine their accuracy by applying them to various topologies under different traffic patterns.

A. Complexity

One of the main aims of this paper was to propose analytical models with reduced complexity to enable the study of large networks. We now study the complexity of the various techniques presented in this paper and also compare them against those presented in [5].

The computational requirements of the Independence model are $O(JC^2)$ for calculation of the β s (6) and $O(C)$ for calculation of path blocking. Note that the complexity of calculation is *independent* of hop length. Moreover, the $O(JC^2)$ for calculation of β can be reduced to $O(C^2)$ by parallel computation. The computational requirements for the Correlation model are the same as those of the Independence Model for the calculation of β s and $O(2HC) + O(C)$ for path blocking calculations since we are computing blocking for a path in both directions. Though no longer independent of hop length, the complexity requirements are still considerably less than those of [5] and [6].

The complexity of computation of route blocking for the technique presented in [5] is $O(C^H)$ for fixed routing, which limits its applicability to small dense networks. We highlight this computational advantage by presenting results of time taken for computation for two networks, the 6-node ring and the 21-node ARPA-2 network, in Table I. All computations were done on a Sun Ultra SPARC system running at 150 MHz and the load was chosen through simulations such that the average network blocking probability was 0.1%. The maximum hop-length was limited to 3 in the 6-node ring, and 4 for the ARPA-2 network. As can be seen from the results, the Independence and Correlation Models are far superior to that of [5] in terms of time complexity. Although, not explicitly evident, the Independence model gives *exactly* the same results as the model in [5]. This is because both models make the same assumptions, and calculate the number of free wavelengths correctly, albeit in different ways, under these assumptions.

Network	Birman's Model (secs)	Independence (secs)	Correlation (secs)
Ring, $C = 8$	3.89	0.06	0.18
Ring, $C = 16$	69.72	0.29	0.84
ARPA-2, $C = 8$	263.02	0.29	3.46
ARPA-2, $C = 16$	2.02×10^4	1.73	6.52

TABLE I

Time complexity of the three models for the ARPA-2 and 6–node ring network. All times are in seconds, and C refers to the capacity of each link.

B. Numerical Results

We now present results of both our techniques for a variety of topologies and compare them against simulations to study their accuracy.

For fixed routing, 4 topologies were chosen, a 6-node ring, a 12-node ring, a 13-node Mesh network (Figure 1), and the 21-node ARPA-2 network (Figure 2). The ring networks were chosen to study the efficacy of our methods when applied to sparse networks, and the Mesh and ARPA-2 were chosen as examples of two arbitrary topologies. Calculations are shown for 32 wavelengths. The maximum hop length was restricted to 3 for the 6-node ring, 6 for the 12-node ring, and 5 for the 13-node Mesh and the 21-node ARPA-2 network. The number of routes considered are 18 in the 6-node ring, 72 in the 12-node ring, 73 in the Mesh network, and 76 routes in the ARPA-2 network. The accuracy of our models were studied under three different traffic patterns. They can be compactly written by the equation

$$T_H = q^{H-1} \cdot T_1 \tag{32}$$

where T_i is the traffic on a i-hop path. Three values of q were chosen:

- $q = 1.0$: Uniform Traffic,
- $q = 0.5$: Traffic dominated by smaller hop routes (low correlation), and
- $q = 1.5$: Traffic dominated by larger hop routes (significant correlation).

For simulations, 4,00,000 calls were taken in each batch and 20 batches were run for each load. The data points reported are the midpoints of the 95% confidence intervals. When using the iterative algorithm for analysis, iterations were stopped when blocking estimates in successive steps differed by less than $\epsilon = 10^{-6}$.

We now discuss our results for each traffic pattern and network. Due to lack of space, we show results of the uniform traffic pattern only for the mesh, ARPA-2 and 12-

node ring with 32 wavelengths, and results of the non - uniform patterns ($q = 0.5$, and $q = 1.5$) only for the 12-node ring and the ARPA-2 Networks for 32 wavelengths. However, results for other networks and wavelengths under these traffic patterns are similar and the observations we make are valid for them also.

For the ARPA-2 network (Figure 3) as well as for the 13-node Mesh network (Figure 6) we observe that the Independence Model gives reasonable estimates for the blocking probability under uniform traffic ($q = 1.0$) since the networks are well connected. The estimates obviously improve when the multi-link traffic is less ($q = 0.5$) as shown for the ARPA-2 network in Figure 4 because of reduced correlation. However, when correlation increases ($q = 1.5$), the results of the Independence model degrade for the ARPA-2 network (Figure 5) indicating that the approximation that sets of wavelengths on adjacent links are independent is no longer a good one. The Correlation Model is seen to give fairly good results for both these topologies under all traffic conditions as can be expected from the original formulation.

Results of the 12-node ring network accentuate this difference in the Correlation and the Independence models in handling wavelength correlation. The Independence model overestimates the blocking for the 12-node ring (Figure 7).The results improve only marginally for the 12-node ring under non-uniform traffic with reduced correlation ($q = 0.5$) (Figure 8) and the estimates are off by more than two orders of magnitude when the correlation increases ($q = 1.5$) (Figure 9) confirming results of previous researchers that sparse networks introduce significant wavelength correlation. The accuracy of the Correlation Model in handling this correlation is confirmed by application to such networks. Under all traffic patterns for both the ring networks it is seen to give reasonable estimates . We hence conclude that the Independence Model gives fair estimates for topologies which are well connected and have traffic patterns that result in low to medium correlation, while the Correlation Model may be used on a wide variety of networks even when connectivity is sparse and traffic patterns induce large correlation.

VI. Fixed Alternate and Least Loaded Routing

We have extended the Independence Model to analyse the Fixed Alternate and Least Loaded Routing schemes. However, we do not present the theory here due to lack of space but instead show two plots for these schemes. In Figure 10 we have plotted the results for the 6-node ring with 16 wavelengths and Fixed Alternate Routing for a reservation parameter[1] of $r = 0$ and observe that the results are reasonably accurate for analytical purposes. In Figure 11 we have plotted results for a 4-node fully connected network with 16 wavelengths and Least Loaded Routing for a reservation parameter of $r = 2$. Again it is seen that the results are fairly accurate. We are still researching the problem of estimating the correlation coefficients for the Correlation Model under these schemes and hence do not show any plots for it.

The computation requirements for Fixed Alternate Routing are $O(SRC^2) + O(JSRC)$ where S is the total number of node pairs and R is the average number of routes for each node pair. This assumes that computations are done using the Independence Model. The computation requirements for Least Loaded Routing are similar to those of [5] because only two hops were considered, although our analysis can clearly be extended to larger number of hops without worsening the complexity.

VII. Conclusions

We have proposed two analytical techniques of low complexity, the Independence Model and the Correlation Model, for the study of wavelength routed networks with arbitrary topology and traffic patterns. Through computations we have shown that the Independence Model gives good estimates when the network is well connected while the Correlation Model is accurate for both sparse and well connected networks under fixed routing. We have also shown, by analysing their complexity and through numerical computation, that these techniques have low computational requirements and are suitable for analysis of large networks. The Independence Model in particular, has a complexity which is *independent* of hop-length. Also, it gives the same estimates as the model in [5], without suffering from the exponential computation bottleneck. The Correlation Model also has low computational cost, but is however not insensitive to the direction in which we proceed along a route when we compute the blocking probability and may lead to incorrect results under highly skewed traffic patterns. We have also extended the Independence Model to study Fixed Alternate Routing and Least Loaded Routing and found it to give reasonable results, though more experiments are required to thoroughly analyse its efficacy.

A possible bottleneck in our techniques is that of round-off errors. In the inclusion-exclusion equation (9), the combinatorial term becomes extremely huge for large capacities (≥ 64 wavelengths) and when multiplied by the

[1] A reservation parameter of r signifi es that a route must have at least $r + 1$ free wavelengths if it is to be used as an alternate route for a call.

probability term can introduce significant round-off errors if the blocking probabilities are small ($\approx 10^{-3}$). This results in blocking probabilities that are negative or greater than 1. Hence we feel that caution must be used when using this technique for analysing networks with very large capacities at very low blocking probabilities. However current networks have a capacity of around 30-40 wavelengths and we feel that our techniques are adequate for analysing them. A possible heuristic for skirting the errors is to set the offending probabilities to zero since they would be extremely small in the first place to have caused such errors. This may result in the iterative procedure failing to converge and can be avoided by reducing the margin of error. We have applied this technique with some success and show the results for the ARPA-2 network in Figure 12 with 64 wavelengths and fixed routing. The iterations were stopped when the blocking estimates in successive steps differed by less than 10^{-4}. As can be seen from Figure 12, the results are reasonably accurate.

Several extensions to our work are possible. An immediate possibility is to extend our technique to include limited-wavelength conversion. Further, techniques are required which can give accurate estimates for networks with first fit wavelength assignment when the number of wavelengths is large. This is especially important, since for large capacities, the first fit would yield much better throughput at low blocking than random wavelength assignment. Another possible direction is to develop more accurate heuristics, or even better, correct expressions for wavelength correlation between adjacent links which is insensitive to the direction in which we trace a route while calculating blocking probabilities.

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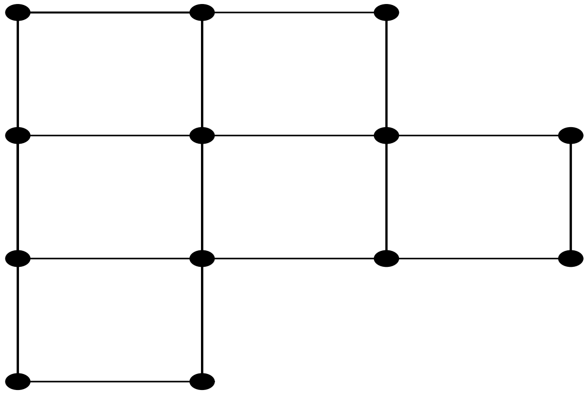

Fig. 1. A 13-node 18-link mesh network

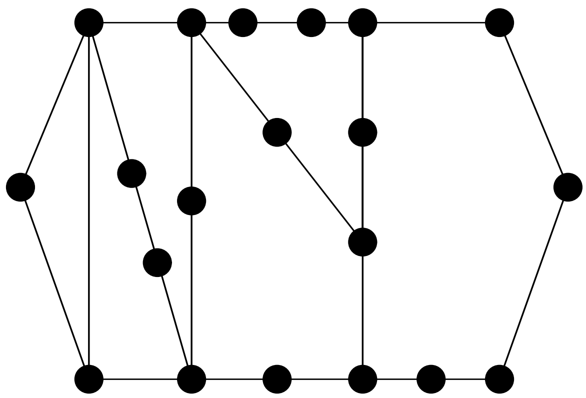

Fig. 2. The 21-node 26-link ARPA-2 network

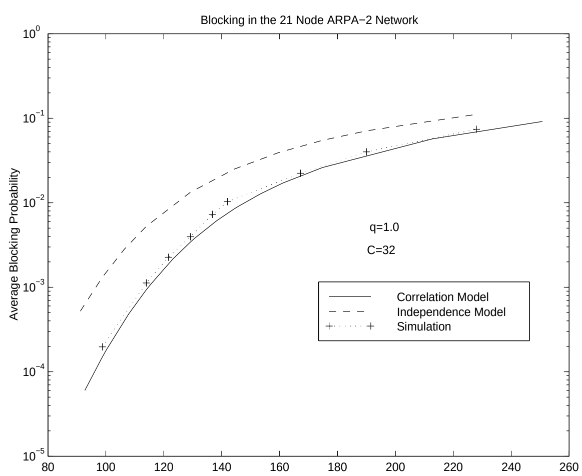

Fig. 3. Plot showing average blocking probability of the ARPA-2 network for C=32 and uniform traffi c

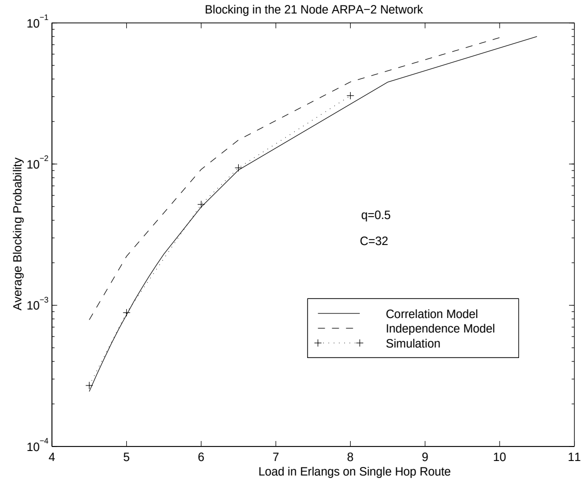

Fig. 4. Plot showing average blocking probability of the ARPA-2 network for C=32 and nonuniform traffi c ($q = 0.5$)

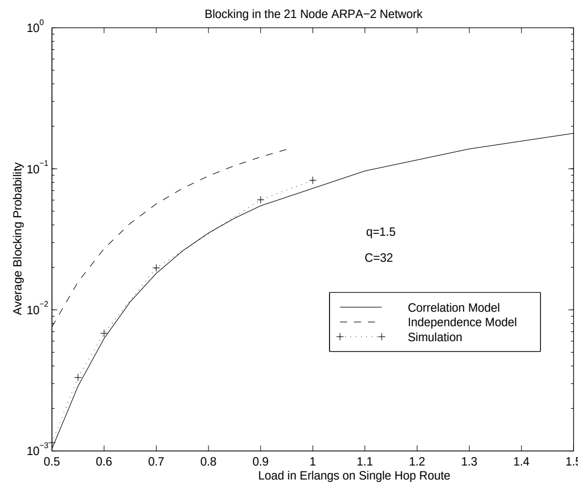

Fig. 5. Plot showing average blocking probability of the ARPA-2 network for C=32 and nonuniform traffi c ($q = 1.5$)

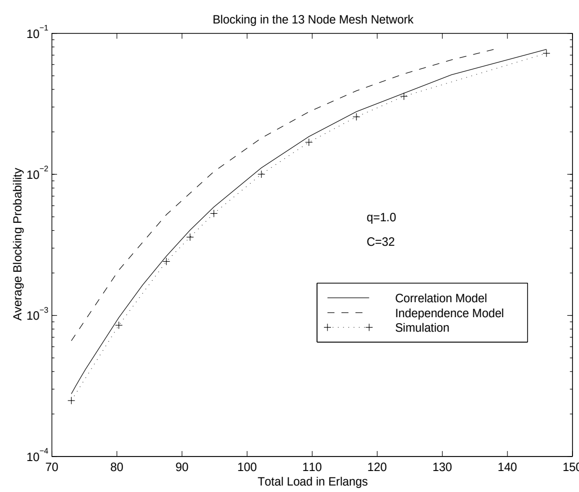

Fig. 6. Plot showing average blocking probability of the 13–node mesh network for $C = 32$ and uniform traffi c

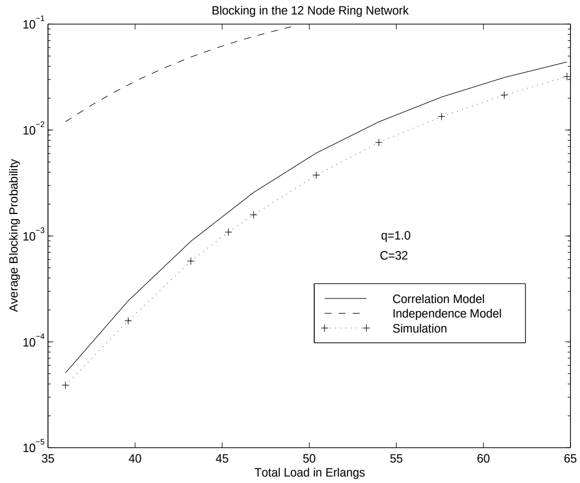

Fig. 7. Plot showing average blocking probability of the 12-node ring network for $C = 32$ and uniform traffic

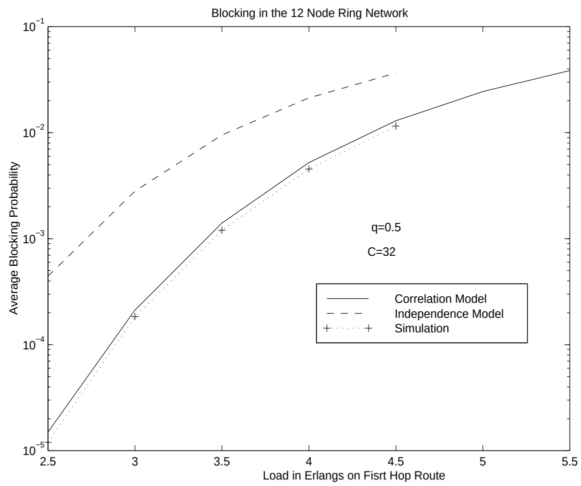

Fig. 8. Plot showing average blocking probability of the 12-node ring network for $C = 32$ and nonuniform traffic ($q = 0.5$)

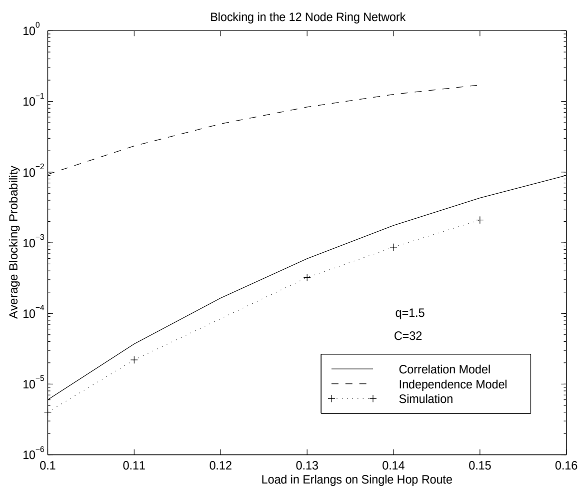

Fig. 9. Plot showing average blocking probability of the 12-node ring network for $C = 32$ and nonuniform traffic ($q = 1.5$)

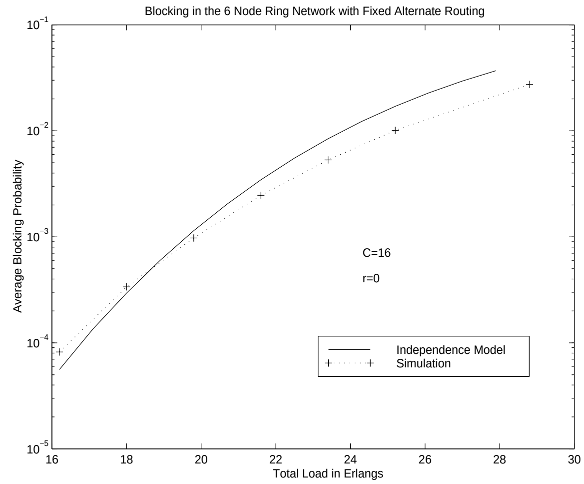

Fig. 10. Plot showing average blocking probability of the 6-node ring network for $C = 16$, and Fixed Alternate Routing ($r = 0$)

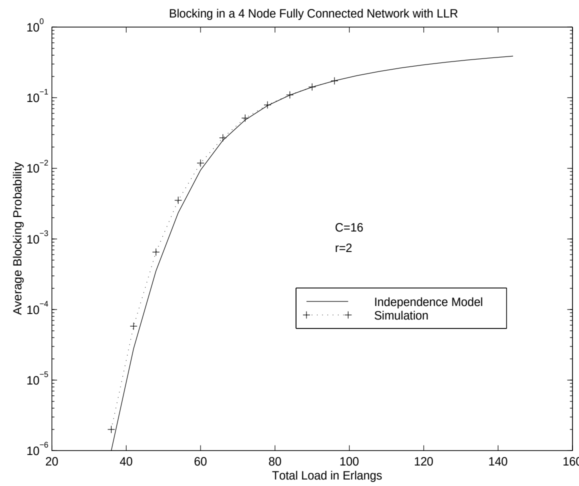

Fig. 11. Plot showing average blocking probability of a 4-node fully connected network for $C = 16$, uniform traffic and Least Loaded Routing ($r = 2$)

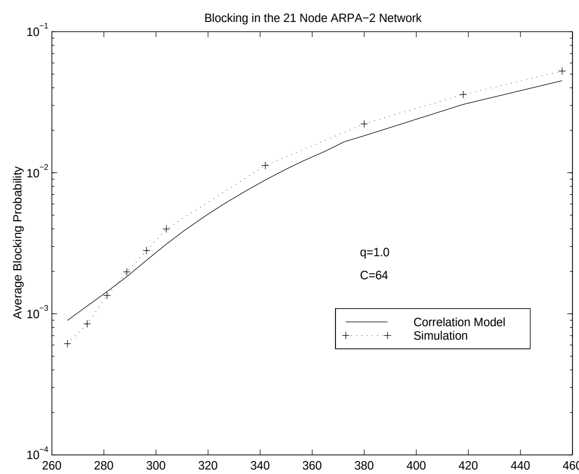

Fig. 12. Blocking in the ARPA-2 network with C=64 and uniform traffic